Week 5

9/23/25

- Made it! Only two problems/week from here on
- Lost / Trouble / Difficult / Understood Everything

Lost	Trouble	Difficult	Understood Everything
$\underbrace{1}_{\text{Let's chat!}}$	$\underbrace{6}_{\text{Let's chat}}$	10	3

- OH Changes: Tomorrow → Thursday 12:30-2:00
- Pytorch learning curve

Review

Classification e.g.,

- spam vs ham
- hand written digits
- breast cancer
- diabetes

Focus on binary (easier visualizations)

① Linear + Sigmoid $f(x) = \sigma(\langle w, x\rangle)$

$$= \frac{1}{1+e^{-\langle w, x\rangle}}$$

② Binary Cross Entropy

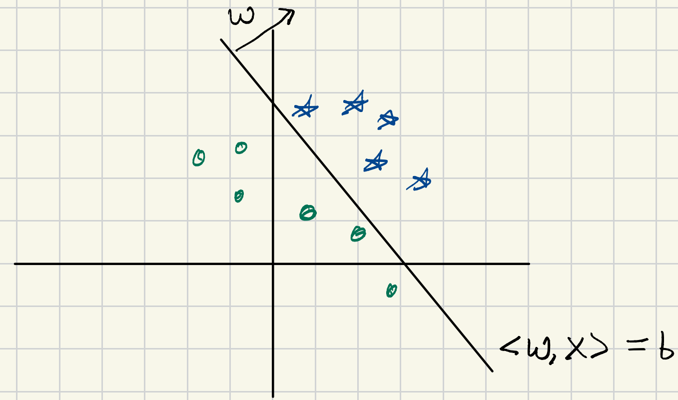
$$y \log(\sigma(\langle w,x\rangle)) + (1-y)\log(1-\sigma(\langle w,x\rangle))$$

③ Gradient Descent

$$\nabla \mathcal{L}(w) = X^T(\sigma(Xw) - y)$$

$$f(x) = \sigma(\langle w, x\rangle) \geq \tau$$

$$\Leftrightarrow \quad \langle w, x\rangle \geq b \quad \text{where } \tau = \sigma(b)$$



In $\mathbb{R}^2$, w, $w^\perp$ form orthonormal basis

$$x = \alpha w + \beta w_\perp$$

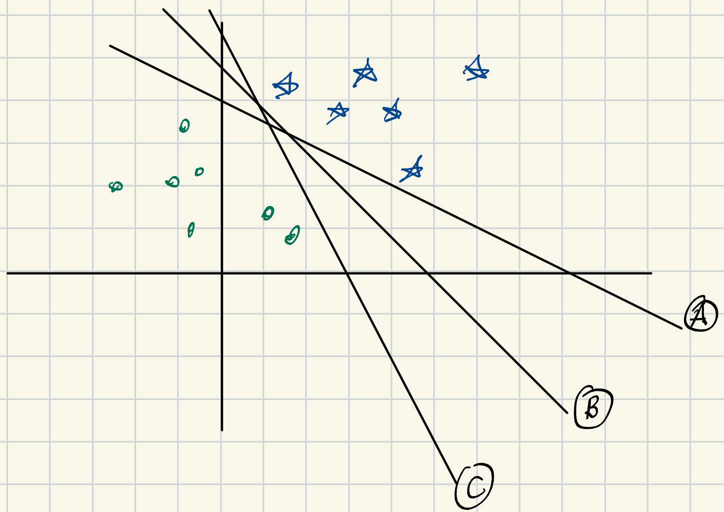
$$\langle x, w\rangle = \alpha\langle w, w\rangle + \beta\langle w_\perp, w\rangle$$
$$= \alpha$$

Support Vector Machines

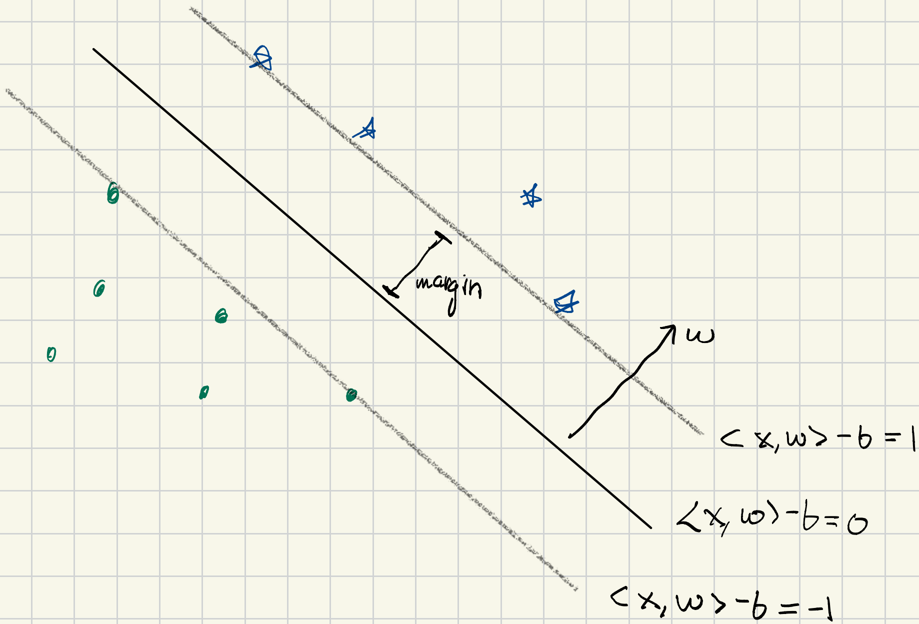
Assumption : linearly separable (extra features help)

$\underline{x}^{(i)} \in \mathbb{R}^d$ $\quad y^{(i)} \in \{-1, 1\}$

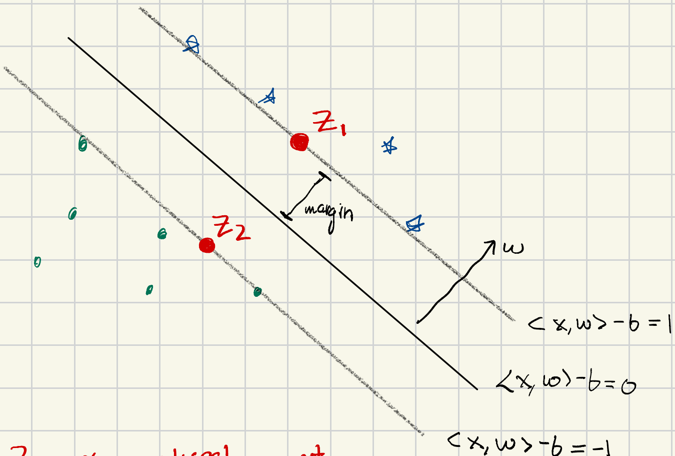
Q: Which separating line to choose?



A: Best margin



Computing the Margin



z_2 is closest point on "lower" hyper plane to z_1

$$\Rightarrow \quad z_1 - z_2 = \lambda \bar{w}$$

where $\bar{w} = \dfrac{w}{\|w\|_2}$

Q: what is $\|\bar{w}\|_2$?

$$
\begin{aligned}
1 &= \langle z_1, w\rangle - b \\
&= \langle \lambda \bar{w} + z_2, w\rangle - b \\
&= \lambda \langle \bar{w}, w\rangle + \langle z_2, w\rangle - b \\
&= \lambda \frac{\langle w, w\rangle}{\|w\|_2} - 1 \\
&= \lambda \|w\|_2 - 1 \qquad \Leftrightarrow \lambda = \frac{2}{\|w\|_2}
\end{aligned}
$$

$$
\begin{aligned}
\text{Goal: } & \max \ \lambda \\
&= \max \frac{2}{\|w\|_2} \\
&= \min \frac{\|w\|_2}{2} \\
&= \min \|w\|_2
\end{aligned}
$$

Constrained Optimization

$$\min_{w,b} \|w\|_2 \quad \text{such that}$$

$$\text{If} \quad y^{(i)} = 1, \quad \langle x^{(i)}, w \rangle - b \geq 1$$

$$\text{If} \quad y^{(i)} = -1, \quad \langle x^{(i)}, w \rangle - b \leq -1$$

$$\Longleftrightarrow \quad \min_{w,b} \|w\|_2 \quad \text{such that}$$

$$y^{(i)} \left[\langle x^{(i)}, w \rangle - b \right] \geq 1 \quad \forall i$$